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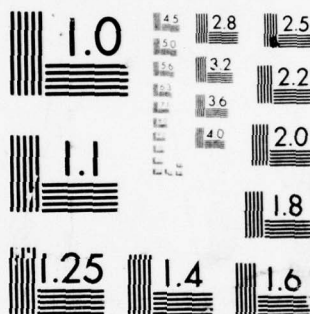
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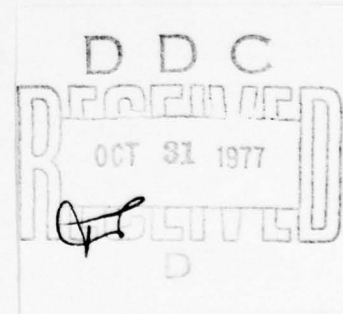
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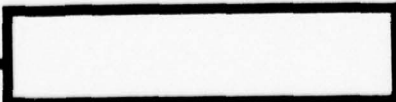
PRANDTL EQUATION IN THE THEORY OF A WING WITH A
FINITE SPAN

by

D. I. Sherman



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PRANDTL EQUATION IN THE THEORY OF A WING WITH A
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By: D. I. Sherman

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|-------|------------|-----------------|-------|------------|-----------------|
| А а | А а | A, a | Р р | Р р | R, r |
| Б б | Б б | B, b | С с | С с | S, s |
| В в | В в | V, v | Т т | Т т | T, t |
| Г г | Г г | G, g | У у | У у | U, u |
| Д д | Д д | D, d | Ф ф | Ф ф | F, f |
| Е е | Е е | Ye, ye; E, e* | Х х | Х х | Kh, kh |
| Ж ж | Ж ж | Zh, zh | Ц ц | Ц ц | Ts, ts |
| З з | З з | Z, z | Ч ч | Ч ч | Ch, ch |
| И и | И и | I, i | Ш ш | Ш ш | Sh, sh |
| Й й | Й й | Y, y | Щ щ | Щ щ | Shch, shch |
| К к | К к | K, k | Ъ ъ | Ъ ъ | " |
| Л л | Л л | L, l | Ы ы | Ы ы | Y, y |
| М м | М м | M, m | Ь ь | Ь ь | ' |
| Н н | Н н | N, n | Э э | Э э | E, e |
| О о | О о | O, o | Ю ю | Ю ю | Yu, yu |
| П п | П п | P, p | Я я | Я я | Ya, ya |

*ye initially, after vowels, and after ъ, ъ; e elsewhere.
 When written as ё in Russian, transliterate as yë or ë.
 The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

| | | | | | | |
|---------|---|---|---|---------|---|-----|
| Alpha | A | α | α | Nu | N | ν |
| Beta | B | β | | Xi | Ξ | ξ |
| Gamma | Γ | γ | | Omicron | Ο | ο |
| Delta | Δ | δ | | Pi | Π | π |
| Epsilon | E | ε | ε | Rho | Ρ | ρ ϑ |
| Zeta | Z | ζ | | Sigma | Σ | σ ς |
| Eta | H | η | | Tau | Τ | τ |
| Theta | Θ | θ | θ | Upsilon | Υ | υ |
| Iota | I | ι | | Phi | Φ | φ φ |
| Kappa | K | κ | κ | Chi | Χ | χ |
| Lambda | Λ | λ | | Psi | Ψ | ψ |
| Mu | M | μ | | Omega | Ω | ω |

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English |
|---------|---------|
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| | |
|-----------|----------------------------|
| sin | sin |
| cos | cos |
| tg | tan |
| ctg | cot |
| sec | sec |
| cosec | csc |
| sh | sinh |
| ch | cosh |
| th | tanh |
| cth | coth |
| sch | sech |
| csch | csch |
| arc sin | \sin^{-1} |
| arc cos | \cos^{-1} |
| arc tg | \tan^{-1} |
| arc ctg | \cot^{-1} |
| arc sec | \sec^{-1} |
| arc cosec | \csc^{-1} |
| arc sh | \sinh^{-1} |
| arc ch | \cosh^{-1} |
| arc th | \tanh^{-1} |
| arc cth | \coth^{-1} |
| arc sch | sech^{-1} |
| arc csch | csch^{-1} |

| | |
|-----|------|
| rot | curl |
| lg | log |

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0185

PRANDTL EQUATION IN THE THEORY OF A WING WITH A FINITE SPAN¹

by D. I. Sherman

Presented by Academician L. S. Leybenzon

Footnote: ¹Given at a seminar of the Institute of Mechanics of the AS USSR in December 1947. End footnote

§1. Prandtl [1] obtained the following singular integro-differential equation for determining circulation in a flow about a wing with a finite span:

$$\frac{8\pi}{mb(x)} \Gamma(x) - \int_{-a}^a \frac{\Gamma(t)}{t-x} dt = 4\pi V \alpha(x). \quad (1)$$

Here $2a$ is the wingspan, $b(x)$ is the chord of its profile and

$\alpha(x)$ its geometric angle of attack, V is the velocity of the air flow at infinity, π is a certain constant (approximately equal to 2π), and $\Gamma(x)$ is the unknown air flow circulation along the wing profile. As usual, the diverging integral on the left side of the equation should be considered in the sense of the main Cauchy value.

Equation (1) has been the object of many studies. This equation was transformed into an integral Fredholm equation with a continuous kernel¹ in a relatively recent study by I. N. Vekua [2].

Footnote: ¹L. G. Magnaradze [3] obtained a similar Fredholm equation with somewhat different assumptions before I. N. Vekua. End footnote

In this case, in accordance with the statement of the problem, I. N. Vekua considers that $b(x)$ has the form

$$b(x) = \frac{V a^2 - x^2}{p(x)} \quad (-a < x < a), \quad (2)$$

where $p(x)$ is an even analytical function in segment $-a < x < a$, which assumes positive values in it. The angle of attack $\alpha(x)$ and unknown circulation $\Gamma(x)$ are also assumed to be even functions in the same segment.

In practice it is extremely significant when $p(x)$ is the rational function

$$p(x) = \frac{\sum_{k=0}^n a_k x^{2k}}{\sum_{k=0}^m b_k x^{2k}} \quad (3)$$

with real coefficients a_k and b_k .

G. Schmidt [4] found a sufficiently effective solution (in squares) for this case (at $b_0 = 0$). Then L. I. Sedov found this solution in a different manner.

I. N. Vekua found that the kernel of the Fredholm equation to which he had reduced equation (1) is degenerate if $p(x)$ is rational. Therefore, using the method he proposed, in this case the solution can also be obtained in finite form.

This report presents a somewhat different approach to solving equation (1) when function $p(x)$ is rational. We feel that this approach may sometimes have certain advantages over the others, since

it can be directly (without using a conformal expression) transferred to the more complex situation in which the segment joining points $t = -a$ and $t = a$ is curvilinear, and coefficients a_k, b_k and the free term in (1) are complex. We encounter this type of complex Prandtl equation in certain problems of the elasticity theory.

In conclusion, we will indicate an approximate method of solving equation (1) which is suitable for any function $b(x)$, and which, in our opinion, can have sufficiently effective results.

We will draw a profile along real segment $(-a, a)$ on plane $z = x + iy$, $\sqrt{a^2 - z^2}$ is arbitrarily used to mean that branch of this function which assumes positive values on the upper bank of the profile¹.

Footnote: ¹Obviously, in this case $\sqrt{a^2 - x^2}$ in formula (2) should be considered to be the limiting value of $\sqrt{a^2 - z^2}$ on the upper bank of the profile. End footnote

For definiteness we will say that $n_1 = n_2 = n$. We can consider the other cases in a similar fashion.

We will introduce the function

$$\Phi(z) = \frac{1}{2\pi i} \int_{-a}^a \frac{\Gamma(t)}{t-z} dt, \quad (4)$$

at regular intervals on plane z everywhere outside of the profile. Then, considering the conditions which are normally accepted in wing theory

$$\Gamma(-a) = \Gamma(a) = 0, \quad (5)$$

we will write equation (1) in the following form, after transformation:

$$\begin{aligned} & \frac{8}{\pi} \sum_{n=0}^{\infty} a_n t^{2n} \left\{ \left(\frac{\Phi(t)}{\sqrt{a^2 - t^2}} \right)_i + \left(\frac{\Phi(t)}{\sqrt{a^2 - t^2}} \right)_e \right\} - \\ & - i \sum_{n=0}^{\infty} b_n t^{2n} \{ \Phi'_i(t) + \Phi'_e(t) \} = 4V_\infty(t) \sum_{n=0}^{\infty} b_n t^{2n}, \quad (6) \\ & \Gamma(t) = \Phi_i(t) - \Phi_e(t) \quad (-a < t < a), \end{aligned}$$

where indices i and e indicate that the limiting values of the functions to which they are assigned are taken on the upper and lower banks of the profile, respectively.

Then setting

$$\Psi(z) = \frac{8}{m} \frac{\Phi(z)}{\sqrt{a^2 - z^2}} \sum_0^n a_k z^{2k} - i\Phi'(z) \sum_0^n b_k z^{2k}, \quad (7)$$

we will have

$$\Psi_i(t) + \Psi_e(t) = f(t), \quad f(t) = 4V_a(t) \sum_0^n b_k t^{2k}. \quad (8)$$

Function $\Psi(z)$ is regular on plane z outside of profile $(-a, a)$, with the exception of an infinitely distant point where it obviously has a pole on the order of $2(n-1)$. We will introduce a new function which is also regular outside the profile, but which returns to zero at infinity:

$$\Psi^*(z) \equiv \Psi(z) - \sum_0^{2(n-1)} c_k^* z^k, \quad (9)$$

where, at suitable values of constants c_k^* the second term on the right side expresses the main part of the expansion (together with the free term) of function $\Psi(z)$ in the vicinity of an infinitely far-off point. Here from (8) we will have

$$\Psi_i^*(t) + \Psi_e^*(t) = f(t) - 2 \sum_0^{2(n-1)} c_k^* t^k. \quad (10)$$

Whence, after simple computations, we will have

$$Y^*(z) = \omega(z) - \sum_0^{2(n-1)} c_k z^k + \frac{1}{\sqrt{a^2 - z^2}} \left(C + \sum_0^{2n-1} c_k z^k \right), \quad (11)$$

whereupon:

$$\omega(z) = \frac{1}{\sqrt{a^2 - z^2}} \frac{1}{2\pi i} \int_{-\sigma}^{\sigma} \frac{\sqrt{a^2 - t^2} f(t) dt}{t - z}, \quad c_k = i \sum_0^{n-E(k)} (-1)^{k_1} c_{\frac{k_1}{2}}^{\frac{k_1}{2}} a^{2k_1} c_{k+2k_1-1}^*, \quad (12)$$

where C is a new random constant and $E(k) = \frac{k}{2} \sim \frac{k+1}{2}$ depending on whether k is even or odd.

Footnote: As formulae (12) show, each of the constants c_k ($k = 1, 3, \dots, 2n-1$) and c_k ($k = 2, 4, \dots, 2(n-1)$) is expressed by all $c_{k_1}^*$ ($k_1 = 0, 2, \dots, 2(n-1)$) and $c_{k_1}^*$ ($k_1 = 1, 3, \dots, 2n-3$), respectively, beginning with $k_1 = k-1$. Constant c_0 depends on the same $c_{k_1}^*$ ($k_1 = 1, 3, \dots, 2n-3$) as c_2 and, therefore, can be expressed by c_k ($k = 2, \dots, 2(n-1)$).

Thus, we will have all $2n$ random constants:

c_k ($k = 1, 2, \dots, 2n-1$) and C . The conditions for finding them are given below. Henceforth constants c_k^* ($k = 0, 1, \dots, 2(n-1)$) will not be necessary.

We note that constant c_{-1} (at $k_1 = k_2 = 0$) in formula (12) should be considered to be equal to zero. End footnote

Including C in constant c_0 and using the previous designations for it, based on formulae (7) and (9) we will have

$$\frac{8}{m} \frac{\Phi(z)}{\sqrt{a^2 - z^2}} \sum_{k=0}^n a_k z^{2k} - i \Phi'(z) \sum_{k=0}^n b_k z^{2k} = \omega(z) + \frac{1}{\sqrt{a^2 - z^2}} \sum_{k=0}^{2n-1} c_k z^k. \quad (13)$$

From equation (13) we will have

$$\Phi(z) = T(z) \left\{ \int_{z_0}^z \frac{Q(t)}{T(t)} dt + c_{2n} \right\}; \quad (14)$$

here c_{2n} is a new random constant and, furthermore, the following designations are introduced

$$\begin{aligned}
 T(z) &= (V a^2 - z^2 - iz)^{\beta_0} \prod_{k=1}^{2n} \left(\frac{V a^2 - z^2 - i(z - a_k) - \gamma_k}{V a^2 - z^2 - i(z - a_k) + \gamma_k} \right)^{\beta_k}, \\
 Q(z) &= \frac{i}{g(z)} \left\{ \omega(z) + \frac{1}{V a^2 - z^2} \sum_{k=0}^{2n-1} c_k z_k^k \right\}, \\
 g(z) &= \sum_{k=0}^n b_k z^{2k},
 \end{aligned} \tag{15}$$

where a_k are the roots to polynomial $g(z)$ which, by definition, lie outside of segment $(-a, a)$ and are assumed to be simple for convenience; $\beta_k (k=0, 1, \dots, 2n)$ are certain constants, the first of which β_0 is real; z_0 is the initial point of integration and¹

$$\gamma_k = V a^2 - a_k^2.$$

Footnote: ¹We should take γ_k to be the value of the selected branch $V a^2 - z^2$ at $z = a_k$. End footnote

§2. Analytical function $T(z)$ will be single-valued on a plane on which $2n$ more other profiles² are drawn in a certain fashion from points a_k along with the above profile along the real segment $(-a, a)$.

Footnote: ²At least one of these profiles should be drawn to an infinitely distant point. End footnote

Here, obviously, the integration curve in (14), which joins z and z_0 , should neither intersect the profiles, nor, generally speaking, contain points α_k .

In accordance with formula (4), we will select the constants $c_k (k=0,1,\dots, 2n)$ so that function $\Phi(z)$ is regular at points $z=\alpha_k$ and returns to zero at infinity.

We will first¹ assume that $\beta_0 > -1$.

Footnote: ¹Based on the assumptions made with respect to coefficients a_k and b_k constant $\beta_0 > 0$. However, here we are only considering the more general case which is of interest in several other problems. End footnote

In this case, setting $c_{2n} = 0$, we will take an infinitely distant point as z_0 . Then

$$\Phi(z) = T(z) \int_{(\infty)}^z \frac{Q(t)}{T(t)} dt. \quad (16)$$

Expanding the right side of this formula into a series at sufficiently large values of modulus z , it is easy to prove that $\Phi(z)$ is single-valued in the vicinity of an infinitely far-off point and returns to zero at infinity.

Further, near $z = \alpha_k$ we will have

$$\begin{aligned} T(z) &= (z - \alpha_k)^{\nu_k} \{ \varepsilon_{0k} + \varepsilon_{1k} (z - \alpha_k) + \dots \}, \\ \frac{Q(z)}{T(z)} &= (z - \alpha_k)^{-(1 + \nu_k)} \{ \delta_{0k} + \delta_{1k} (z - \alpha_k) + \dots \}, \\ \frac{\delta_k}{\gamma_k} &= \nu_k, \end{aligned} \quad (17)$$

where ε_{jk} and δ_{jk} are certain constants.

If $\operatorname{Re}(\nu_k) < 0$ ($k = 1, \dots, 2n$), then in order for function $\Phi(z)$ to be regular at points $z = \alpha_k$, the following equalities must be observed

$$\int_{(\infty)}^{\alpha_k} \frac{Q(t)}{T(t)} dt = 0 \quad (k = 1, \dots, 2n). \quad (18)$$

We will allow that this system can be solved relative to constants $c_k (k = 0, 1, \dots, 2n-1)$. Then, determining the latter from it and substituting their values in (16), we will find the required function $\Phi(z)$.

Considering $\beta_0 > -1$ like before, we will examine the case in which certain of $\operatorname{Re}(v_k) > 0$. We will say that $\operatorname{Re}(v_k) = n_k + \mu_k$, where n_k is the largest whole number contained in the real part of v_k and $0 < \mu_k < 1$. It is easy to see that the corresponding equations in (18) will look different in this case.

Substituting formula (16) in form

$$\Phi(z) = T(z) \left[\int_{(\infty)}^z \left\{ \frac{Q(t)}{T(t)} - \sum_{j=0}^{n_k} \frac{\delta_{jk}}{(t - \alpha_k)^{v_k - j + 1}} \right\} dt + \sum_{j=0}^{n_k} \frac{\delta_{jk}}{(t - \alpha_k)^{v_k - j}} \right] \quad (19)$$

on the basis of (17) and observing that the function included under the sign of the integral in these equations is absolutely integrable at $z = \alpha_k$, instead of certain of (18) we will obtain the following equations which provide regularity of $\Phi(z)$ at the points in question:

$$\int_{(\infty)}^{\infty} \left\{ \frac{Q(t)}{r(t)} - \sum_{j=0}^{n_k} \frac{v_j}{(t-a_k)^{v_k-j+1}} \right\} dt = 0. \quad (20)$$

If $\text{Im}(v_k) = \mu_k = 0$, for any value of k , the corresponding equation from the latter must be replaced by condition $\delta_{n,k} = 0$.

Now we will proceed to the case of $\beta_0 < -1$, for brevity limiting ourselves to the assumption that $\text{Re}(v_k) < 0 (k=1, \dots, 2n)$. Let $\beta_0 = -(n_0 + \mu_0)$, where n_0 is a positive integer and $0 < \mu_0 < 1$.

Taking $z_0 = 0$ in (14) as the initial point of integration in this case, instead of (18) we will have the following equations (also following from the condition of regularity of $\Phi(z)$ at the points in question):

$$\int_0^{\infty} \frac{Q(t)}{r(t)} dt + c_{2n} = 0 \quad (k=1, \dots, 2n). \quad (21)$$

Subsequently, designating A as a fixed number with a rather large modulus and considering the expansion

$$\frac{Q(z)}{r(z)} = \sum_{k=0}^{\infty} \frac{v_k}{z^k + \beta_k + 2}$$

in the vicinity of an infinitely distant point, where ϵ_k are certain constants, we will rewrite (14) as follows

$$\begin{aligned} \Phi(z) = T(z) & \left[\int_0^A \frac{Q(t)}{r(t)} dt + \int_A^{(\infty)} \left\{ \frac{Q(t)}{r(t)} - \sum_{k=0}^{n_0-1} \frac{\epsilon_k}{t^{2+k+\beta_0}} \right\} dt + \right. \\ & + \int_{(\infty)}^z \left\{ \frac{Q(t)}{r(t)} - \sum_{k=0}^{n_0-1} \frac{\epsilon_k}{t^{2+k+\beta_0}} \right\} dt - \sum_0^{n_0-1} \frac{\epsilon_k}{(1+k+\beta_0) s^{1+k+\beta_0}} + \\ & \left. + \sum_0^{n_0-1} \frac{\epsilon_k}{(1+k+\beta_0) A^{1+k+\beta_0}} + c_{2n} \right]. \end{aligned} \quad (22)$$

Whence it is clear that in order for $\Phi(z)$ to return to zero at infinity, the following equation must be added to system (21)

$$\int_0^A \frac{Q(t)}{r(t)} dt - \int_{(\infty)}^A \left\{ \frac{Q(t)}{r(t)} - \sum_{k=0}^{n_0-1} \frac{\epsilon_k}{t^{2+k+\beta_0}} \right\} dt + \sum_0^{n_0-1} \frac{\epsilon_k}{(1+k+\beta_0) A^{1+k+\beta_0}} + c_{2n} = 0. \quad (23)$$

Determining constants $\epsilon_k (k=0, 1, \dots, 2n)$, from them, we will find $\Phi(z)$.

In particular, at $\mu_0 = 0$ it is necessary to require observation of equality $\epsilon_{n_0-1} = 0$ instead of (23).

As we saw, function $\Phi(z)$, was regular in the vicinity of an infinitely far-off point in all the cases in question. Furthermore, under the conditions (18), (20) or (21), (23) (and the partial changes in them), which we will consider to be satisfied for each case, $\Phi(z)$ does not have peculiarities on the plane with profile $(-a, a)$. Consequently, it is single-valued on it. Whence it follows that the integral in (14), taken for segment $(-a, a)$, should be equal to zero. The latter, in turn, makes it possible to easily determine that the unknown function $\Gamma(f)$, determined by $\Phi(z)$ on the basis of (6), satisfies equations (5).

Note. In conclusion we will briefly discuss the case when $p(t)$ is any analytical function in segment $(-a, a)$. We will say that function $p(t)$ assumes identical values at the geometrically coinciding points on the upper and lower banks of profile $(-a, a)$. Then, using the conformal reflection of the plane with the same profile on the circumference of a single circle, after certain computations we will have

$$p(t) = \sum_{k=0}^{\infty} a_k \omega_k(t), \quad a_k = -\frac{1}{2\pi} \int_{-\sigma}^{\sigma} \frac{p(t) \omega_k(t)}{\sqrt{t^2 - a^2}} dt, \quad (k=0, 1, \dots, \infty),$$

$$\omega_0 = 1, \quad \omega_k(t) = \left\{ \left(\frac{t + \sqrt{t^2 - a^2}}{a} \right)^k + \left(\frac{t - \sqrt{t^2 - a^2}}{a} \right)^k \right\} (k=1, \dots, \infty), \quad (24)$$

where $\sqrt{t^2 - a^2}$ is an imaginary positive number on the upper bank of the profile.

Now we will designate $\chi(t)$ as the sum $n+1$ of the first terms (including the first term) of series (24) and we will assume that function $p(t)$ is approximately equal to $\chi(t)$ (which is permissible in practice, beginning with a certain n). Noting that here $\chi(t) = \chi(z)$, we will transform equation (1) into a form analogous to (8), setting

$$\Psi(z) = \frac{8}{\pi} \chi(z) \frac{\Phi(z)}{\sqrt{a^2 - z^2}} - i\Phi'(z). \quad (25)$$

From this point on we continue analogously to the above procedure.

Institute of Mechanics of the AS USSR

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| A205 DMATC | 1 | E053 AF/INAKA | 1 |
| A210 DMAAC | 2 | E017 AF/ RDXTR-W | 1 |
| B344 DIA/RDS-3C | 8 | E404 AEDC | 1 |
| C043 USAMIIA | 1 | E408 AFWL | 1 |
| C509 BALLISTIC RES LABS | 1 | E410 ADTC | 1 |
| C510 AIR MOBILITY R&D | 1 | E413 ESD | 2 |
| LAB/FIO | | FTD | |
| C513 PICATINNY ARSENAL | 1 | CCN | 1 |
| C535 AVIATION SYS COMD | 1 | ETID | 3 |
| C557 USAIIC | 1 | NIA/PHS | 1 |
| C591 FSTC | 5 | NICD | 5 |
| C619 MIA REDSTONE | 1 | | |
| D008 NISC | 1 | | |
| H300 USAICE (USAREUR) | 1 | | |
| P005 ERDA | 2 | | |
| P055 CIA/CRS/ADD/SD | 1 | | |
| NAVORDSTA (50L) | 1 | | |
| NAVWPNSCEN (Code 121) | 1 | | |
| NASA/KSI | 1 | | |
| 544 IES/RDPO | 1 | | |
| AFIT/LD | 1 | | |

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